19097

```
sage: A = ([1, 1], [3, 1], [-1, -1])
sage: b = (1000, 1500, -400)
sage: c = (10, 5)
sage: P = InteractiveLPProblemStandardForm(A, b, c)
sage: D = P.dictionary(1, 3, 4)
```

```
D
```

$$\begin{array}{|rcrcrcr|}
\hline
x_1 & = & 400 & - & x_2 & + & x_5 \\
x_3 & = & 600 & & & - & x_5 \\
x_4 & = & 300 & + & 2x_2 & - & 3x_5 \\
\hline
z & = & 4000 & - & 5x_2 & + & 10x_5 \\
\hline
\end{array}$$

```
D.run_simplex_method()
```

$$\begin{array}{|rcrcrcr|}
\hline
x_1 & = & 400 & - & x_2 & + & x_5 \\
x_3 & = & 600 & & & - & x_5 \\
x_4 & = & 300 & + & 2x_2 & - & 3x_5 \\
\hline
z & = & 4000 & - & 5x_2 & + & 10x_5 \\
\hline
\end{array}$$

Entering: x_5. Leaving: x_4.

$$\begin{array}{|rcrcrcr|}
\hline
x_1 & = & 500 & - & \frac{1}{3}\,x_2 & - & \frac{1}{3}\,x_4 \\
x_3 & = & 500 & - & \frac{2}{3}\,x_2 & + & \frac{1}{3}\,x_4 \\
x_5 & = & 100 & + & \frac{2}{3}\,x_2 & - & \frac{1}{3}\,x_4 \\
\hline
z & = & 5000 & + & \frac{5}{3}\,x_2 & - & \frac{10}{3}\,x_4 \\
\hline
\end{array}$$

Entering: x_2. Leaving: x_3.

$$
\begin{array}{|rcl|}
\hline
x_1 &=& 250 + \frac{1}{2}x_3 - \frac{1}{2}x_4 \\
x_2 &=& 750 - \frac{3}{2}x_3 + \frac{1}{2}x_4 \\
x_5 &=& 600 - x_3 \\
\hline
z &=& 6250 - \frac{5}{2}x_3 - \frac{5}{2}x_4 \\
\hline
\end{array}
$$

```
P.run_simplex_method2()
```

$$
\begin{array}{|rcl|}
\hline
x_3 &=& 1000 - x_1 - x_2 \\
x_4 &=& 1500 - 3x_1 - x_2 \\
x_5 &=& -400 + x_1 + x_2 \\
\hline
z &=& 0 + 10x_1 + 5x_2 \\
\hline
\end{array}
$$

The initial dictionary is infeasible, solving auxiliary problem.

$$
\begin{array}{|rcl|}
\hline
x_3 &=& 1000 + x_0 - x_1 - x_2 \\
x_4 &=& 1500 + x_0 - 3x_1 - x_2 \\
x_5 &=& -400 + x_0 + x_1 + x_2 \\
\hline
w &=& 0 - x_0 \\
\hline
\end{array}
$$

Entering: x_0. Leaving: x_5.

$$
\begin{array}{|rcl|}
\hline
x_3 &=& 1400 + x_5 - 2x_1 - 2x_2 \\
x_4 &=& 1900 + x_5 - 4x_1 - 2x_2 \\
x_0 &=& 400 + x_5 - x_1 - x_2 \\
\hline
w &=& -400 - x_5 + x_1 + x_2 \\
\hline
\end{array}
$$

Entering: x_1. Leaving: x_0.

$$
\begin{array}{|rcl|}
\hline
x_3 &=& 600 - x_5 + 2x_0 \\
x_4 &=& 300 - 3x_5 + 4x_0 + 2x_2 \\
x_1 &=& 400 + x_5 - x_0 - x_2 \\
\hline
w &=& 0 - x_0 \\
\hline
\end{array}
$$

Back to the original problem.

$$
\begin{array}{|rcl|}
\hline
x_3 &=& 600 - x_5 \\
x_4 &=& 300 - 3x_5 + 2x_2 \\
x_1 &=& 400 + x_5 - x_2 \\
\hline
z &=& 4000 + 10x_5 - 5x_2 \\
\hline
\end{array}
$$

Entering: x_5. Leaving: x_4.

$$
\begin{array}{|rcl|}
\hline
x_3 &=& 500 + \frac{1}{3} x_4 - \frac{2}{3} x_2 \\
x_5 &=& 100 - \frac{1}{3} x_4 + \frac{2}{3} x_2 \\
x_1 &=& 500 - \frac{1}{3} x_4 - \frac{1}{3} x_2 \\
\hline
z &=& 5000 - \frac{10}{3} x_4 + \frac{5}{3} x_2 \\
\hline
\end{array}
$$

Entering: x_2. Leaving: x_3.

$$
\begin{array}{|rcl|}
\hline
x_2 &=& 750 + \frac{1}{2} x_4 - \frac{3}{2} x_3 \\
x_5 &=& 600 - x_3 \\
x_1 &=& 250 - \frac{1}{2} x_4 + \frac{1}{2} x_3 \\
\hline
z &=& 6250 - \frac{5}{2} x_4 - \frac{5}{2} x_3 \\
\hline
\end{array}
$$

The optimal value: 6250. An optimal solution: $(250,\ 750)$.

```
P.run_simplex_method()
```

$$
\begin{array}{|rcl|}
\hline
x_3 &=& 1000 - x_1 - x_2 \\
x_4 &=& 1500 - 3x_1 - x_2 \\
x_5 &=& -400 + x_1 + x_2 \\
z &=& 0 + 10x_1 + 5x_2 \\
\hline
\end{array}
$$

The initial dictionary is infeasible, solving auxiliary problem.

$$
\begin{array}{|rcl|}
\hline
x_3 &=& 1000 + x_0 - x_1 - x_2 \\
x_4 &=& 1500 + x_0 - 3x_1 - x_2 \\
x_5 &=& -400 + x_0 + x_1 + x_2 \\
\hline
w &=& 0 - x_0 \\
\hline
\end{array}
$$

Entering: x_0. Leaving: x_5.

$$
\begin{array}{|rcl|}
\hline
x_3 &=& 1000 + x_0 - x_1 - x_2 \\
x_4 &=& 1500 + x_0 - 3x_1 - x_2 \\
x_5 &=& -400 + x_0 + x_1 + x_2 \\
\hline
w &=& 0 - x_0 \\
\hline
\\
\hline
x_3 &=& 1400 + x_5 - 2x_1 - 2x_2 \\
x_4 &=& 1900 + x_5 - 4x_1 - 2x_2 \\
x_0 &=& 400 + x_5 - x_1 - x_2 \\
\hline
w &=& -400 - x_5 + x_1 + x_2 \\
\hline
\end{array}
$$

Entering: x_1. Leaving: x_0.

$$
\begin{array}{|rcl|}
\hline
x_3 &=& 1400 + x_5 - 2x_1 - 2x_2 \\
x_4 &=& 1900 + x_5 - 4x_1 - 2x_2 \\
x_0 &=& 400 + x_5 - x_1 - x_2 \\
\hline
w &=& -400 - x_5 + x_1 + x_2 \\
\hline
\\
\hline
x_3 &=& 600 - x_5 + 2x_0 \\
x_4 &=& 300 - 3x_5 + 4x_0 + 2x_2 \\
x_1 &=& 400 + x_5 - x_0 - x_2 \\
\hline
w &=& 0 - x_0 \\
\hline
\end{array}
$$

Back to the original problem.

$$
\begin{array}{|rcl|}
\hline
x_3 &=& 600 - x_5 \\
x_4 &=& 300 - 3x_5 + 2x_2 \\
x_1 &=& 400 + x_5 - x_2 \\
\hline
z &=& 4000 + 10x_5 - 5x_2 \\
\hline
\end{array}
$$

Entering: x_5. Leaving: x_4.

$$
\begin{array}{|rcl|}
\hline
x_3 &=& 600 - x_5 \\
x_4 &=& 300 - 3x_5 + 2x_2 \\
x_1 &=& 400 + x_5 - x_2 \\
\hline
z &=& 4000 + 10x_5 - 5x_2 \\
\hline
\\
\hline
\end{array}
$$