

unstable incomplete gamma

January 18, 2016

The incomplete gamma isn't stable in the real domain:

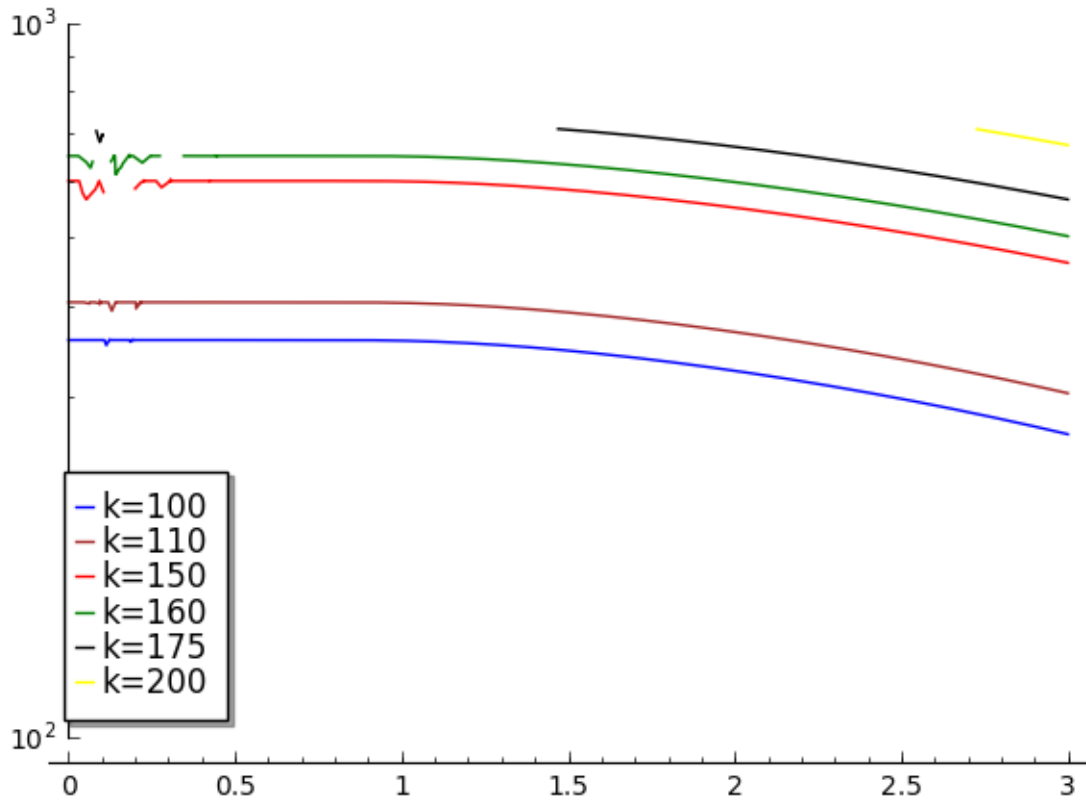
```
In [15]: %display latex
         var('k')
         R10 = RealField(1000)
         f(k, x) = log(gamma(k, x * k))
         f
```

```
Out[15]: (k, x) |--> log(gamma(k, k*x))
```

```
In [16]: def plot1(f): return sum(
         plot_semilogy(
             ## float: see http://trac.sagemath.org/ticket/18210
             f(R10(k)),
             (x, R10(0), R10(3)),
             color=color,
             legend_label='k=%i' % k,
             figsize=6,
             aspect_ratio=1.0,
         )
         for k, color in zip(
             [100, 110, 150, 160, 175, 200],
             ['blue', 'brown', 'red', 'green', 'black', 'yellow', 'orange', 'cyan', 'purple', 'magenta']
         )
         )
         plot1(f)
```

```
verbose 0 (2733: plot.py, generate_plot_points) WARNING: When plotting, failed to evaluate function at 9
verbose 0 (2733: plot.py, generate_plot_points) Last error message: 'can't convert complex to float'
verbose 0 (2733: plot.py, generate_plot_points) WARNING: When plotting, failed to evaluate function at 1
verbose 0 (2733: plot.py, generate_plot_points) Last error message: 'can't convert complex to float'
```

```
Out[16]:
```



There's an obvious bite out of that plot. The correct output is smooth and real.
 A 3d plot of the same:

```
In [17]: def plot2(f): return plot3d(f, (x, 0, 5/2), (k, 100, 200))
         plot2(f)
```

Out[17]: Graphics3d Object

Again, there's an obvious bite out of the function.

Within that domain, the output is *negative*, when it shouldn't be, in the real, positive domain. If I take the absolute value, the output is closer to correct, but still wrong:

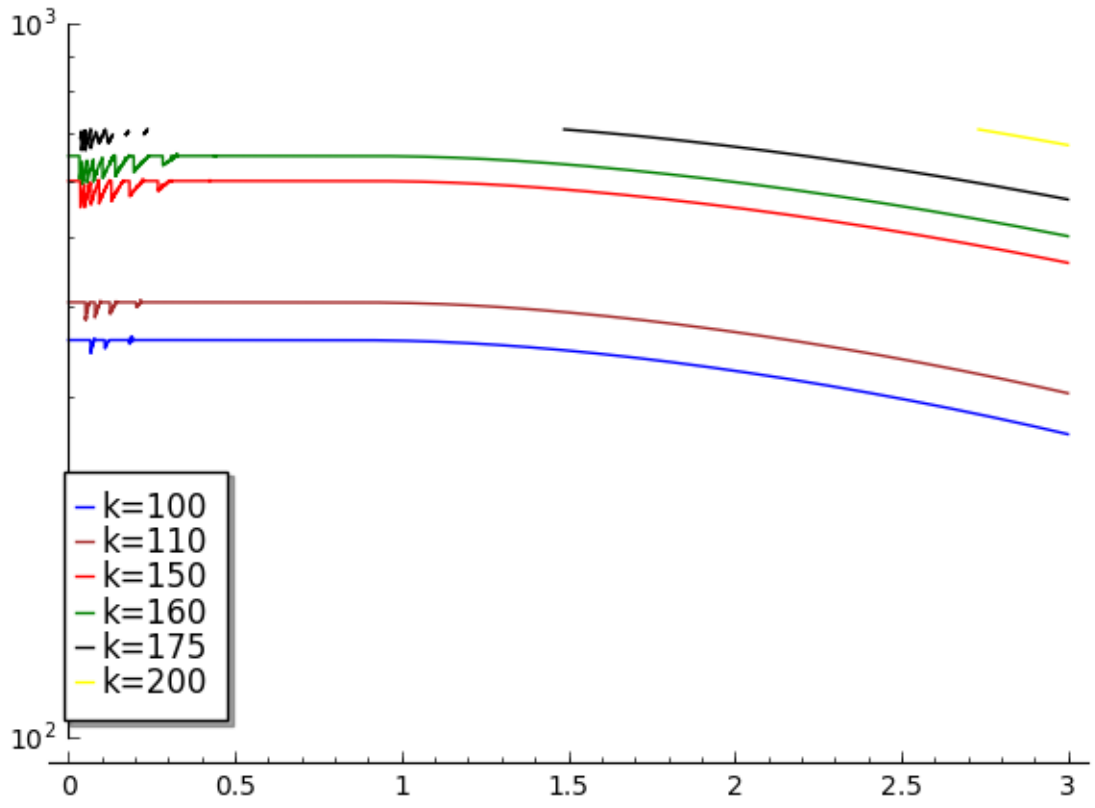
```
In [18]: g = log(abs(gamma(k, x*k)))
         g
```

Out[18]: log(abs(gamma(k, k*x)))

```
In [19]: plot1(g)
```

```
verbose 0 (2733: plot.py, generate_plot_points) WARNING: When plotting, failed to evaluate function at 8
verbose 0 (2733: plot.py, generate_plot_points) Last error message: ''
verbose 0 (2733: plot.py, generate_plot_points) WARNING: When plotting, failed to evaluate function at 1
verbose 0 (2733: plot.py, generate_plot_points) Last error message: ''
```

Out[19]:



In [20]: plot2(g)

Out[20]: Graphics3d Object

In []: